

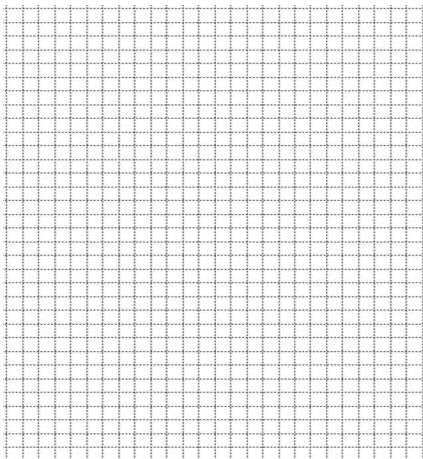
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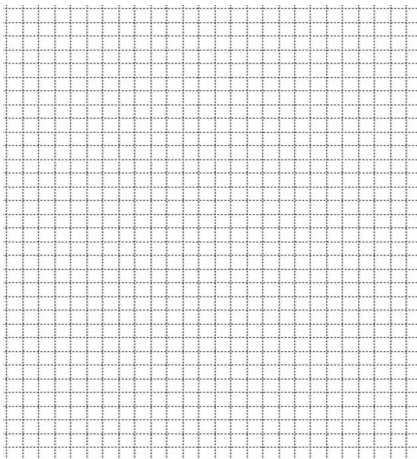
Math 12 Honours: Section 5.4 Graphing Exponential & Logarithmic Equations with Transformations

1. For each graph below, find the Y-intercept, X-intercept (if any), Domain and Range, and Asymptotes. Then graph the function with the grid provided. Be sure to label the axis on your grid.

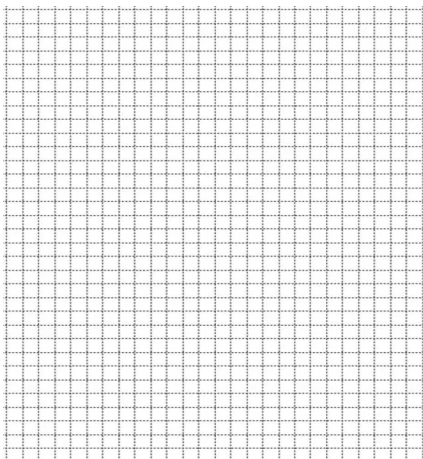
a) $y = 3(2)^{x-2} - 3$



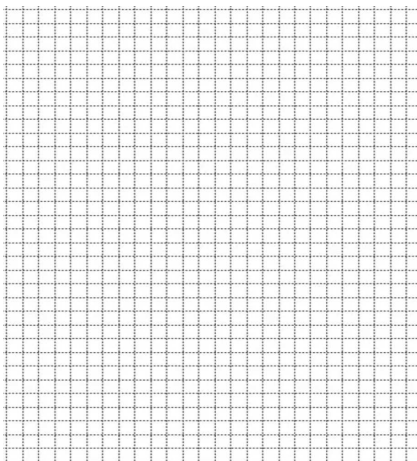
b) $y = -0.5(3)^{-x} + 1$



c) $y = -2(0.33)^{2-x} + 4$

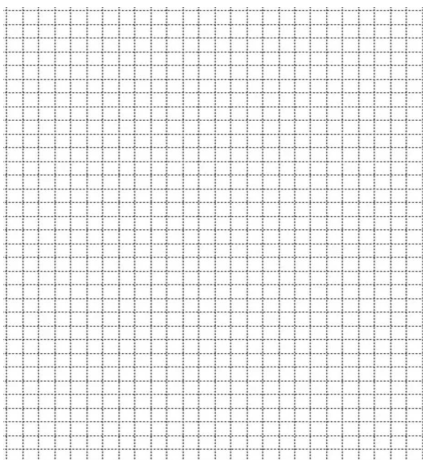


d) $y = 4(81)^{0.25x-1}$

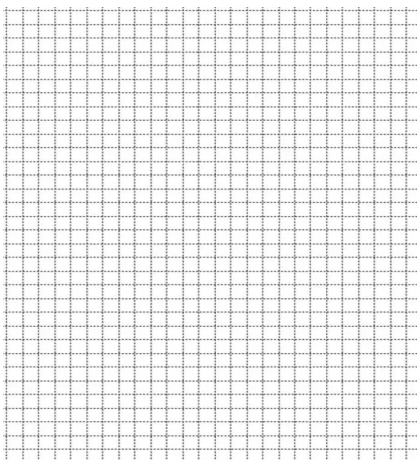


2. Graph each of the following logarithmic functions. Indicate the Domain, Range, equations of Asymptotes, and any intercepts. Be sure to label your axis on the graph:

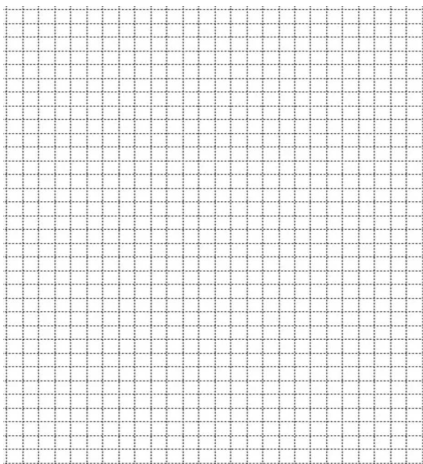
a) $y = \log(2x - 3) + 1$



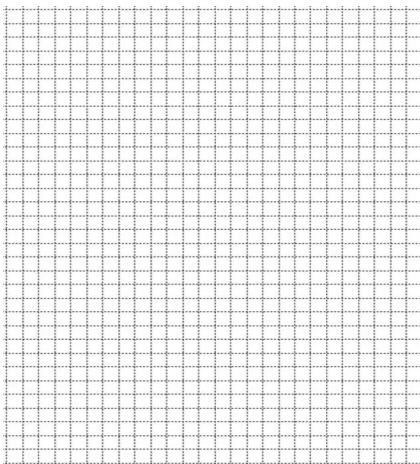
b) $y = \log(4x + 1) - 3$



$$c) y = \log_2(2 - x) + 4$$

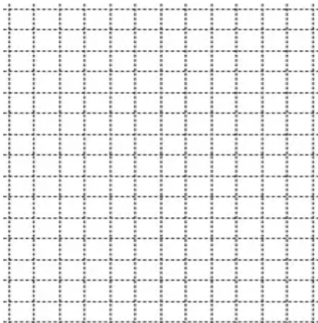


$$d) \log = -\log_3(3x) - 5$$

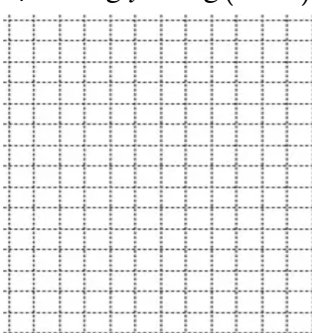


3. Graph the following functions. Indicate the domain and range:

$$a) \log y = 2 \log x$$



$$b) 0.5 \log y = \log(x - 2)$$



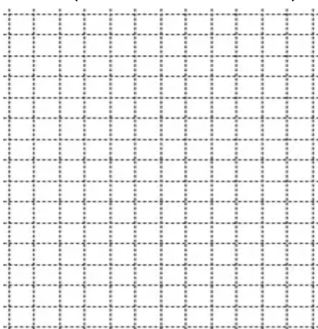
$$c) \log y = \log(\sin x)$$



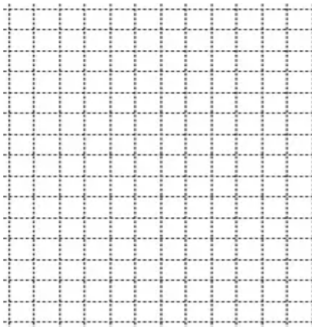
$$d) \log y = \log(\sec x)$$



$$\log_y(x^3 + 3x^2 + 3x + 1) = 3$$



$$y = \log(\log x)$$



4. Given that $f(x) = \log_3(x + 2) - 4$, find $f^{-1}(x)$, the inverse function of $f(x)$

5. What transformation is required to go from $y = \log x$ to $y = \log\left(\frac{1}{x}\right)$?
6. What transformation is required to go from $y = \log_3 x$ to $y = \log_3 \frac{4}{x}$?
7. Are the following graphs the same? Yes or NO? Explain:
- a) $y = 4(0.5)^x$ and $y = 4(2)^{-x}$ b) $y = 24(0.5)^{2-x}$ and $y = 6(2)^x$
8. What is the inverse function of $f(x) = 3(5)^{x-2}$? What are the domain, range, x-intercepts, and Y-intercepts of both $f(x)$ and $f^{-1}(x)$? What patterns do you notice?
9. Find the coordinates of the points of intersection of the graphs: $y = \log_{10}(x-2)$ and $y = 1 - \log_{10}(x+1)$ (Euclid)
10. Determine all the points where the two functions intersect: $y = \log_{10} x^4$ and $y = (\log_{10} x)^3$ (Euclid)
11. If $\log(a+b) = x$ and $\log(a^2 - ab + b^2) = y$, then what is the value of $\sqrt[4]{a^3 + b^3}$ in terms of "x" and "y"?

12. Solve the inequality

a) $\log_4 (2x - 3) > 2$

b) $\log_{\frac{1}{4}} x > 5$

13. What is the domain of the following functions:

i) $y = \log_{0.5} (\log_5 x)$

ii) $y = \log_{0.5} \left(\log_5 \left(\log_{\frac{1}{3}} x \right) \right)$

14. Solve for "x": $\log_4 x - \log_x 16 = \frac{7}{6} - \log_x 8$ (Euclid)

15. Find all the values of "x" such that: $\log_{2x} (48\sqrt[3]{3}) = \log_{3x} (162\sqrt[3]{2})$ (Euclid)

16. Determine all real numbers "x" for which: $2\log_2 (x - 1) = 1 - \log_2 (x + 2)$ (Euclid)

17. Determine all pairs of angles (x,y) with $0^\circ \leq x < 180^\circ$ and $0^\circ \leq y < 180^\circ$ that satisfy the following systems of equations: (Euclid)

$$\log_2 (\sin x \cos y) = -\frac{3}{2} \quad \text{and} \quad \log_2 \left(\frac{\sin x}{\cos y} \right) = \frac{1}{2}$$

18. Determine all pairs (a,b) of real numbers that satisfy the following systems of equations: Give your answer in simplified exact form: (Euclid)

$$\sqrt{a} + \sqrt{b} = 8 \quad \text{and} \quad \log_{10} a + \log_{10} b = 2$$

19. Solve for "x" $\log_{2^x} 3^{20} = \log_{2^{x+3}} 3^{2020}$ (AIME)

Problem

The value of x that satisfies $\log_{2^x} 3^{20} = \log_{2^{x+3}} 3^{2020}$ can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

Solution

Let $\log_{2^x} 3^{20} = \log_{2^{x+3}} 3^{2020} = n$. Based on the equation, we get $(2^x)^n = 3^{20}$ and $(2^{x+3})^n = 3^{2020}$. Expanding the second equation, we get $8^n \cdot 2^{xn} = 3^{2020}$. Substituting the first equation in, we get $8^n \cdot 3^{20} = 3^{2020}$, so $8^n = 3^{2000}$. Taking the 100th root, we get $8^{\frac{n}{100}} = 3^{20}$. Therefore, $(2^{\frac{3}{100}})^n = 3^{20}$, and using the our first equation ($2^{xn} = 3^{20}$), we get $x = \frac{3}{100}$ and the answer is 103. ~rayfish